

NAG Toolbox for MATLAB

g05ya

1 Purpose

To generate multi-dimensional quasi-random sequences with a uniform probability distribution.

2 Syntax

```
[quasi, iref, ifail] = g05ya(fcall, seq, iskip, idim, iref)
```

3 Description

Low discrepancy (quasi-random) sequences are used in numerical integration, simulation and optimization. Like pseudo-random numbers they are uniformly distributed but they are not statistically independent, rather they are designed to give more even distribution in multidimensional space (uniformity). Therefore they are often more efficient than pseudo-random numbers in multidimensional Monte Carlo methods.

g05ya generates a set of points x^1, x^2, \dots, x^N with high uniformity in the S -dimensional unit cube $I^S = [0, 1]^S$. One measure of the uniformity is the discrepancy which is defined as follows:

Given a set of points $x^1, x^2, \dots, x^N \in I^S$ and a subset $G \subset I^S$, define the counting function $S_N(G)$ as the number of points $x^i \in G$. For each $x = (x_1, x_2, \dots, x_S) \in I^S$, let G_x be the rectangular S -dimensional region

$$G_x = [0, x_1) \times [0, x_2) \times \dots \times [0, x_S)$$

with volume x_1, x_2, \dots, x_S . Then the discrepancy of the points x^1, x^2, \dots, x^N is

$$D_N^*(x^1, x^2, \dots, x^N) = \sup_{x \in I^S} |S_N(G_x) - Nx_1, x_2, \dots, x_S|.$$

The discrepancy of the first N terms of such a sequence has the form

$$D_N^*(x^1, x^2, \dots, x^N) \leq C_S (\log N)^S + O((\log N)^{S-1}) \quad \text{for all } N \geq 2.$$

The principal aim in the construction of low-discrepancy sequences is to find sequences of points in I^S with a bound of this form where the constant C_S is as small as possible.

g05ya generates the low-discrepancy sequences proposed by Sobol, Faure and Niederreiter.

4 References

Bratley P and Fox B L 1988 Algorithm 659: Implementing Sobol's Quasirandom Sequence Generator *ACM Trans. Math. Software* **14** (1) 88–100

Fox B L 1986 Algorithm 647: Implementation and Relative Efficiency of Quasirandom Sequence Generators *ACM Trans. Math. Software* **12** (4) 362–376

5 Parameters

5.1 Compulsory Input Parameters

1: **fcall** – logical scalar

The type of operation to perform.

fcall = true

The first call for initialization, and there is no output via array **quasi**.

fcall = false

The sequence has been initialized by a prior call to g05ya with **fcall = true**. Random numbers are output via array **quasi**.

2: **seq – string**

The type of sequence to generate.

seq = 'S'

A Sobol sequence.

seq = 'N'

A Niederreiter sequence.

seq = 'F'

A Faure sequence.

Constraint: **seq** = 'S', 'N' or 'F'.

3: **iskip – int32 scalar**

The number of terms in the sequence to skip on initialization.

iskip = 0

All the terms of the sequence are generated.

iskip = k

The first k terms of the sequence are ignored and the first term of the sequence now corresponds to the k th term of the sequence when **iskip** = 0.

If **seq** = 'F', **iskip** is not referenced.

Constraint: if **seq** = 'N' or 'S' and **fcall** = **true**, **iskip** \geq 0.

4: **idim – int32 scalar**

the number of dimensions required.

Constraint: $1 \leq \mathbf{idim} \leq 40$.

5: **iref(2000) – int32 array**

On entry/on exit: workspace used to contain information between calls to the function. The contents of this array should not be changed.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **quasi(idim) – double array**

The random numbers.

If **fcall** = **false**, **quasi**(k) contains the random number for the k th dimension.

2: iref(2000) – int32 array

On entry/on exit: workspace used to contain information between calls to the function. The contents of this array should not be changed.

3: ifail – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g05ya may return useful information for one or more of the following detected errors or warnings.

ifail = 1

On entry, **iskip** < 0, if **seq** = 'N' or 'S' and **fcall** = **true**,
 or **idim** < 1,
 or **idim** > 40,
 or **seq** ≠ 'F', 'N' or 'S'.

ifail = 2

On entry, the array **iref** has not been correctly initialized.

ifail = 3

The value of **iskip** is too large.

ifail = 4

There have been too many calls in the sequence.

ifail = 5

An internal error has occurred within the function. Please contact NAG.

7 Accuracy

Not applicable.

8 Further Comments

The maximum length of the generated sequences is $2^{29} - 1$, this should be adequate for practical purposes. In the case of the Neiderreiter generator g05ya jumps to the appropriate starting point, while for the Sobol generator it simply steps through the sequence. In consequence the Sobol generator with large values of **iskip** will take a significant amount of time.

9 Example

```
fcall = true;
seq = 'S';
iskip = int32(0);
idim = int32(15);
iref = zeros(2000, 1, 'int32');
[quasi, irefOut, ifail] = g05ya(fcall, seq, iskip, idim, iref)

quasi =
    0
    0
    0
    0
```

```
0
0
0
0
0
0
0
0
0
0
0
0
0
0
irefOut =
    array elided
ifail =
    0
```
